# Urban Water Demand Forecasting Using the Stochastic Nature of Short Term Historical Water Demand and supply Pattern

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***Abstract*- Today's big city water utility companies are experiencing high level of water loss due to various problems in covering a large scale of water supply pipeline networks, therefore any significant improvement of water loss prevention from supply network to treatment plant would require an apprehends stochastic nature of historical water demand and supply pattern. For this reason urban water demand forecasting is one of key important parameters used when water utility companies are trying to find more efficient and robust ways of supplying water for a large number of urban consumers. Water demand forecasting also plays a significant role in managing and planning water supply operations and water conservation and optimization strategies.**

**However, traditional forecasting approaches based on a set of deterministic design capacity factors or using demand forecasting algorithms without evaluating the relationship between supply reliability in response to the stochastic nature of historical water consumption data and supply pattern often become misleading due to the inability to sufficiently forecast forthcoming events and lack of relevant historical pattern and data. This paper presents an analysis and water demand forecasting demonstration using the stochastic nature of short term historical water demand and supply Pattern for Lille University Z6 pipeline networks research area in France.**

***Keywords- Urban Water Demand; Time series (AR1); Forecasting; Stochastic Simulation; Historic Demand Pattern***

## I. INTRODUCTION

### *A. The Role of Short Term Water Demand Forecasting*

In the context of operation and maintenance management, short term forecasting water demand or water consumption not only plays a vital role for the water utility companies which are trying to find more efficient ways to supply water, for a large number of households, commercial and institutional businesses, but also contributes to control water losses due to leakage from the distribution pipe networks. Short term urban water demand forecasting system also support water distribution operations, as well as budgeting and financing management, and program tracking and evaluation [2]. The purpose of demand forecasting and modelling also varies from the simplest historical extrapolation to sophisticated analytical models; therefore the choice of an appropriate modelling ought to address the purpose of the forecasting needed by water utility companies, and the quality and quantity of data.

### *B. Overview of Forecasting Approach Using the Stochastic Nature of Short-Term Historical Water Demand and Supply Pattern*

One of the practical applications of time series analysis and stochastic modelling is short term forecasting. In recent years different forecasting methods have been developed, which could be used to control water distribution operation system nearly in real time based on time series data that are collected sequentially overtime period.

In general, the time series of water demand or consumption historical data extracted from SCADA or using different data gathering mechanisms can be divided into deterministic and stochastic components. The deterministic component is one that can be determined for predictive purpose, the stochastic component, however, is constituted by irregular oscillations and random effects which cannot strictly be accounted for physically and which require probabilistic concepts for description have been explained by [6, 14]. This character can be identified by time series analysis and modelled by a suitable autoregressive Integrated Moving Average processes which can be described in terms of (p,_d,_q) notation, where p indicates the autoregressive order, q the moving average order, and d the degree of differencing necessary to achieve stationarity [14].

## II. MODELLING WATER DEMAND DETERMINISTIC COMPONENTS

The modelling of the deterministic components of the time series water demand model consists of periodic components such as seasonal, weekly and hourly historical patterns and persistence components called short-term memory. The periodic components of the time series water demand or consumption data can be modelled by fitting a harmonic series to the annual series of average metered or observed demand of each day of 365 days based on a short-term, pattern-based forecasting approach developed by [1, 4, 7, 15], and by using equation of [6].

*A. The Daily Periodic Component*

Therefore, the long-term average daily water demand $\overline{Q}_d^{D,S}$ pattern is modelled using a Fourier series:

$$\overline{Q}_d^{D,S} = \overline{Q} + \sum_{k=1}^{k}[a_k \cos\frac{2\pi k}{365}d + b_k \sin\frac{2\pi k}{365}d] \tag{1}$$

Where: - $d$ = 1, 2, and 3….365, $\overline{Q}$ is the mean value of seasonal cycle, $k$ = is the number of harmonic considered, $a_k$ and $b_k$ are Fourier Coefficients and calculated as:

$$a_k = \frac{2}{365}\sum_{d=1}^{365}\overline{Q}_d^{D,matr} \cos\frac{2\pi d}{365}k \tag{2}$$

$$b_k = \frac{2}{365}\sum_{d=1}^{365}\overline{Q}_d^{D,metr} \sin\frac{2\pi d}{365}k \tag{3}$$

Where:

$\overline{Q}_d^{D,metr}$ , are the observed average daily water demand.

*B. The Weekly Periodic Component*

The weekly pattern of the deterministic components of the water demand or consumption is based on a set of Weekly Correction Factors representing the weekly periodic component of $\overline{P}_{i,j}^{D,W}$ for each week and for each month and is calculated as:

$$\overline{P}_{i,j}^{D,W} = \overline{Q}_{i,j}^{D} - \overline{Q}_j^{w} \tag{4}$$

Where:

$\overline{Q}_{i,j}^{D}$ is the mean value of the average daily water demand observed on day (Monday…Sunday) $i$ of the week ($i$ = 1, 2, and 3… 7), the season (winter, spring, summer, fall $j$ of the year $j$ = 1… 4)

$\overline{Q}_j^{w}$ is the mean value of the average weekly demand in season $j$ ( which is winter, spring, summer, fall $j$ of the year $j$ = 1… 4)

*C. The Hourly Periodic Component*

The hourly module, of water demand is a time series forecasting model of hourly water consumption for 24h, the model consists of two modules-daily and hourly. Like the daily module the hourly prediction or forecasting is formulated as a set of equations representing the effects a periodic and persistence component. Therefore the hourly water demand of $Q_{t+y}^{h,for}$ forecasted for the time $t$ hours for $y$ hours ahead is determined using

$$Q_{t+1}^{h,for} = Q_d^{D,for} + \overline{P}_{n,i,j}^{h} + \varepsilon_{t+1} \tag{5}$$

Where: - $Q_d^{D,for}$ = the average daily water demand forecasted using Equation (1)

$\overline{P}_{n,i,j}^{h}$ = the hourly deviation representing the daily pattern which is

$$\overline{P}^{h}_{n,i,j} = \overline{Q}^{h}_{n,i,j} - \overline{Q}^{D}_{i,j} \tag{6}$$

Where

$\overline{Q}^{h}_{n,i,j}$ = represents the mean value of the average hourly water demand measured in hour $n$ ( $n$=1, 2, 3… 24, the hour of the day $i$ in season $j$)

$\overline{Q}^{D}_{i,j}$ = is the mean value of the average daily water demand measured on day $i$ , in season $j$ .

*D. Modelling Water Demand Stochastic Components*

The stochastic components of the water demand data are featured by a series of correlating characteristics which represent the short-term memory components. These characteristics can be analyzed using time series and modelled by appropriate autoregressive-moving-average model (ARMA). Therefore the deviation between the average daily water demand and the mean value estimated solely on the basis of the periodic components $\overline{Q}^{D,S}_{d}$ and, $\overline{P}^{h}_{n,i,j}$ is modelled using the autoregressive process AR (1) [1].

$$\sigma^{D}_{d} = \sigma^{D}_{d} = \Phi_1 \cdot \sigma^{D}_{d-1} \tag{7}$$

Where:

$\sigma^{D}_{d}$ is the deviation between the average daily water demand $\overline{Q}^{D}_{d}$ and the mean value estimated based on the basis of the periodic components.

$\overline{Q}^{D,S}_{d}$ and, $\overline{P}^{D,W}_{i,j}$ ➔ ( $i$, $j$ shows both Monday – Friday day 1…7, and winter – summer i.e.1,..4)

$\Phi$ is Auto-regressive parameter can be calibrated based on the basis of the observed deviation, in autoregressive (ARMA) processes of order p.

$$\sigma^{D,metr}_{d} = \overline{Q}^{D,metr}_{d} - [\overline{Q}^{D,S}_{d} + \overline{P}^{D,W}_{i,j}] \tag{8}$$

The daily and hourly residual or persistence component are modelled by using AR (1) on errors $\varepsilon_{t+y-1}$ and $\varepsilon_{t+y-24}$

$$\varepsilon_{t+y} = \Psi_1 \cdot \varepsilon_{t+y-1} + \Psi_{24} \cdot \varepsilon_{t+y-24} \tag{9}$$

The coefficients $\Psi_1$ and $\Psi_{24}$ depends on the hour of the day $t+y \equiv n = 1, 2, \cdots, 24$ $t+y$ is starting from the beginning of the year and calibrated on the bases of the observed (measured) error $\varepsilon^{metr}_{t}$ where:

$$\varepsilon^{metr}_{t} = Q^{h,metr}_{t} - (Q^{D,metr}_{d} + \overline{P}^{h}_{n,i,j}) \tag{10}$$

*E. Daily and Hourly Demand–Forecasting*

Based on the above analysis the average estimated mean daily water demand and the hourly water demand can be calculated using the following equations

$$Q^{D,for}_{d} = \overline{Q}^{D,S}_{d} + \overline{P}^{D,W}_{i,j} + \sigma^{D}_{d} \tag{11}$$

Where:- $\overline{Q}^{D,S}_{d}$ is the long-term average daily water demand representing the seasonal (s) periodic component.

$\overline{P}^{D,W}_{i,j}$ is the correction representing the weekly periodic component.

$\sigma^{D}_{d}$ is representing the medium term persistence component.

The hourly module, of water demand is a time series forecasting model of hourly water consumption for 24h, the model consists of two modules-daily and hourly. Like the daily module the hourly prediction or forecasting is formulated as a set of equations representing the effects a periodic and persistence component. Therefore the hourly water demand of $Q_{t+y}^{h,for}$ forecasted for the time $t$ hours for $y$ hours ahead is determined using

$$Q_{t+1}^{h,for} = Q_d^{D,for} + \overline{P}_{n,i,j}^{h} + \varepsilon_{t+1} \tag{12}$$

Where: - $Q_d^{D,for}$ is the average daily water demand forecasted using Equation (1)

$\overline{P}_{n,i,j}^{h}$ is the hourly deviation representing the daily pattern which is

$$\overline{P}_{n,i,j}^{h} = \overline{Q}_{n,i,j}^{h} - \overline{Q}_{i,j}^{D} \tag{13}$$

Where: - $\overline{Q}_{n,i,j}^{h}$ is represents the mean value of the average hourly water demand measured in hour ( $n$=1, 2, 3… 24, the hour of the day $i$ in season $j$)

$\overline{Q}_{i,j}^{D}$ is the mean value of the average daily water demand measured on day $i$ , in season $j$ .

The rest of this paper discusses the result and analysis of the Zone 6 research project area.

## III. ZONE-SIX (Z6) RESEARCH AREA -LILLE UNIVERSITY

### A. *Introduction*

The University Lille 1 was established in 1854 in Lille, although its academic roots dated back to 1562. It later moved to Villeneuve d'Ascq in 1967, with 25,000 full-time students plus 15,000 students in continuing education (2011). 1,310 permanent faculty members plus 1,200 staff and around 140 CNRS researchers work there in the different University Lille 1 institutes and 43 research labs. University Lille 1 is a member of the European Doctoral College Lille-Nord-Pas de Calais, which produces 400 doctorate dissertations every year. The university is ranked in the world top 200 universities in mathematics by the Shanghai ranking. (News on Lille 1's webpage, http://www.univ-lille1.fr/Accueil/Actualites?id=26313)

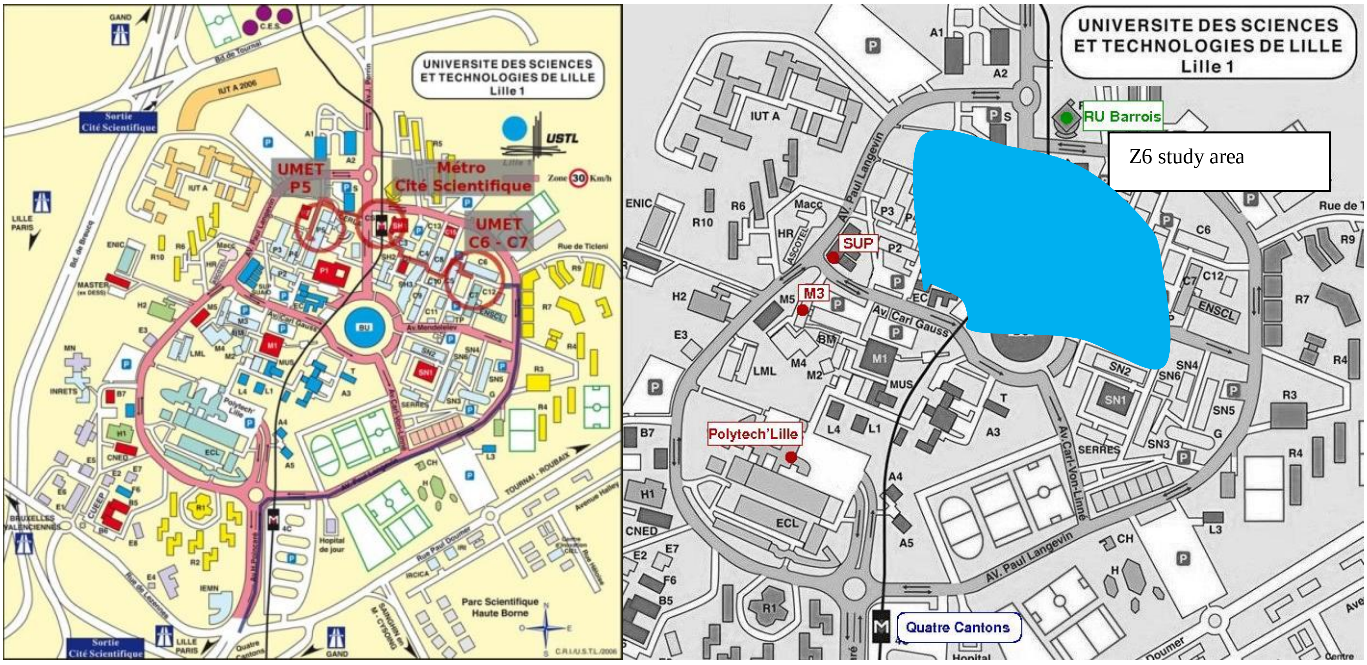



Fig. 1 General and project research area map for Zone-6 map

### B. *Lille University's Water Network System*

The Lille University, drinking water network is divided into different supply zones Z1, Z2, etc. Since the university is planning to implement water loss and leakage prevention by using District Metered Area Zone -6 water demand data have been used for this research purposes. The present water consumption at Zone -6 is about 570 m$^3$ per day, with operating pressure close to 4, 5 bars [13]. The pipe network has about 50 years of age and it is made of cast iron pipes, 150 mm in diameter. The seasonal water demand pattern of the time series (spring, summer, fall and winter) of Lille University is shown in Figure 3, while Figure 2 shows the yearly water demand data pattern.

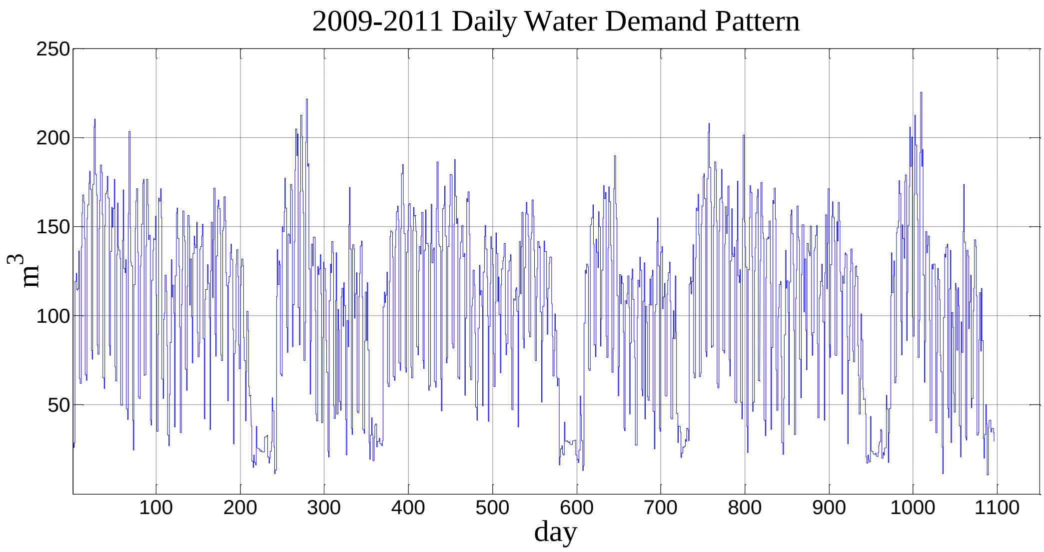
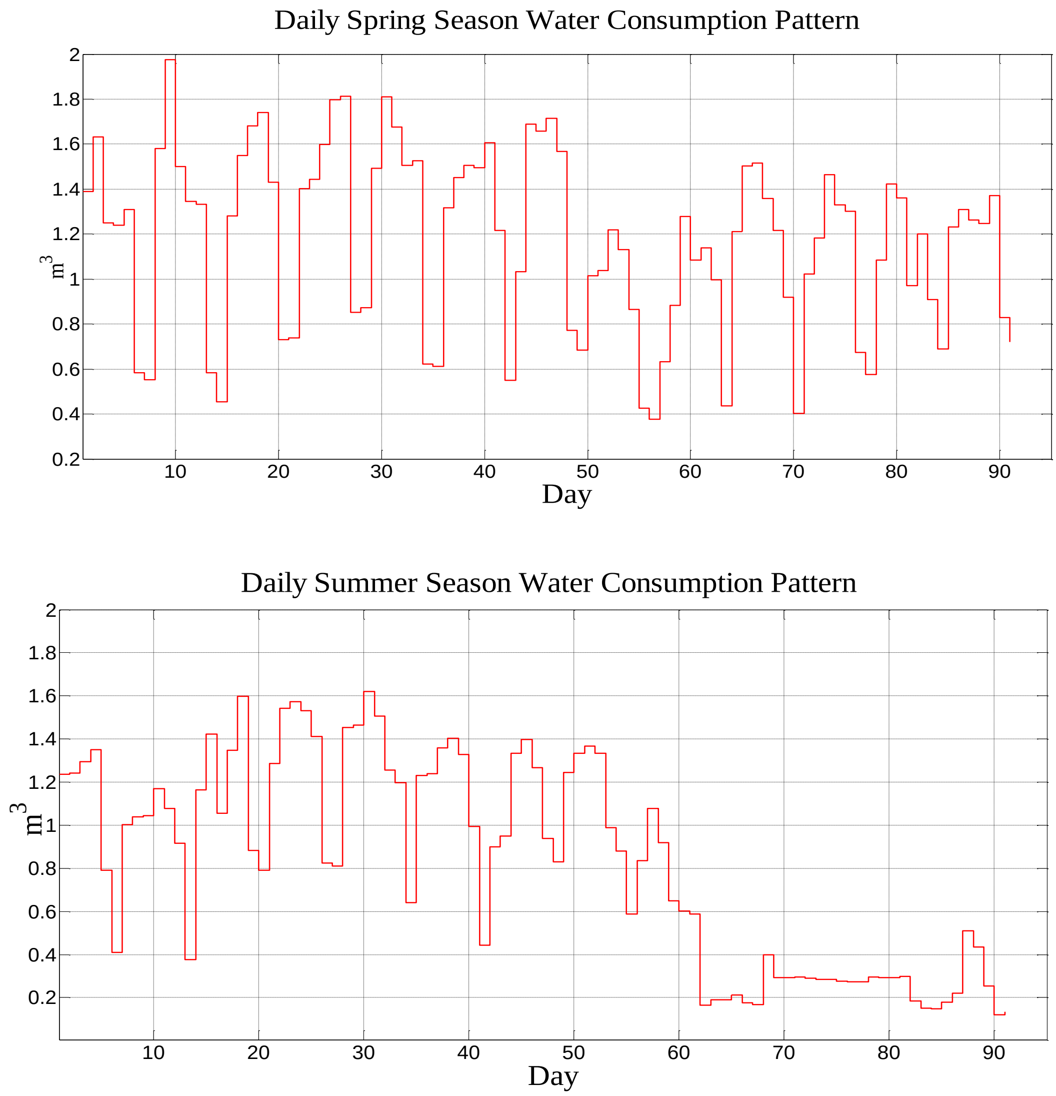
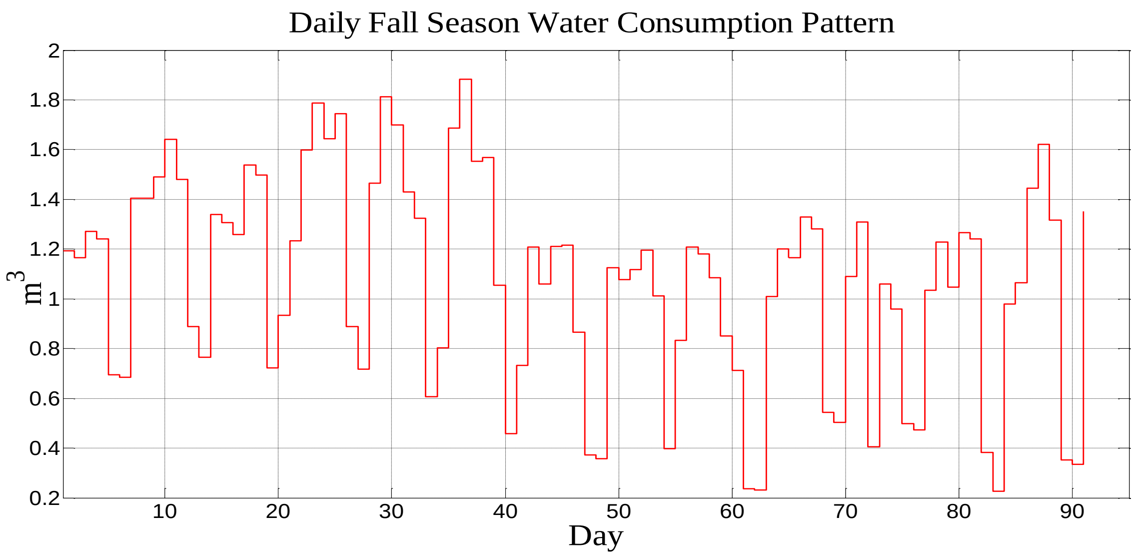



Fig. 2 Three years water demand pattern of the time series





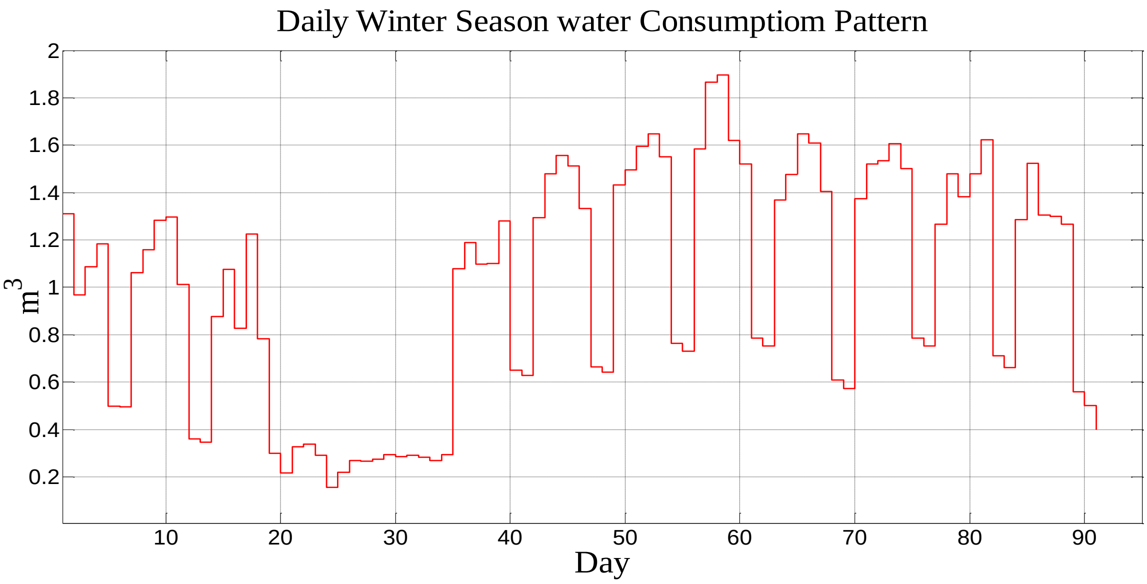




Fig. 3 Seasonal water demand pattern of the time series (spring, summer, fall and winter)

### *C. Model Data*

A stochastic water demand model for synthetic generation of water demands has been formulated and applied in modelling and forecasting water demand for zone 6 water supply Zone project. The model is based on statistical analysis of historical water demand data using a short-term, pattern-based model for water demand forecasting approach [1]. The model comprises a deterministic component and a stochastic component. The deterministic components are modelled by a series of seasonal, weekly and daily patterns. The stochastic components are based on time-series forecasting models that take the short-term memory effect and the random residual components into consideration.

The forecasting and modelling of water demand require reliable data. Reliable data play a key role in the analysis, monitor, and forecast of water system behaviors as bad quality data may result in an erroneous decision scheme [10]. The data employed for zone -6 study area consist of hourly water demand in meter cube per day ($m^3/d$). The water demand data were available for a period of 2009 to 2011; however, during the evaluation of the water demand data for the completeness, correctness process to avoid questionable data and erroneous we found that some of the data were scattered with a missing records, as a result, those missing data have been adjusted and some of them have been completely omitted and the remaining corrected water demand data were considered for model development and testing. All the data were divided into two sets: modelling set consisting of the 2009-2010 years of data, and a testing set consisting of the remaining 2011 year of data. The overall task for this research involves the illustration and analysis of Water Demand Forecasting Using the Stochastic Nature of Short Term Historical Water Demand and Supply Pattern methodologies for Lille University zone-6 using the 2009-2011 water demand / consumption data. This includes an evaluation of considered forecasting methodologies including graphical plots of forecasted vs. observed, time series plots, and relevant statistical measures (e.g., correlation coefficient, residual analyses...etc.).

*D. Forecasting Results Analysis and Discussions*

This study demonstrates how Water Demand Forecasting Using the Stochastic Nature of Short Term Historical Water Demand and Supply Pattern model is useful to study and forecast short term water demand for water utility companies. This paper also demonstrates the data analysis results obtained and shown graphically in Figures 4-5 which confirmed that the model is doing considerably well and gives reliable forecasts for daily and hourly basis of the water demand. The modelling results also indicate that the synthetic patterns, fluctuations and statistics of the generated demands are consistent with the actual ones.

To measure the forecasting accuracy Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Tracking signals (TR) were used and the result on Table 1 shows the summary of analysis. Based on the calculations results of accuracy and sensitivity measures the MAE is 3.28% for daily and 1.63% for hourly respectively the ideal MAE is zero which would mean there is no forecasting error, the larger the MAE, the less the accurate the resulting model. The Tracking signal (TR) is also calculated to measure how often our estimations have been above or below the actual value and to decide where to re-evaluate. Based on the analysis made the TR value has been found that 0.54 for daily and 0.85 for hourly water demand respectively. Most of the Positive Tracking signal values show the actual values are above forecasted values, on the other hand, the Negative Tracking signal values shows the actual values are below the forecasted values, and in most of the time it ranges from 4 and -4. $(4 \leq \mathrm{TR} \leq -4)$. The squared correlation coefficient or the coefficient of determination has been calculated to evaluate the strength of a relationship between forecasted and observed water demand data, based on the analysis it has been found that 0.98 for daily and 0.97 for hourly respectively and this value shows there is strong relation between the forecasted and observed water demand value.

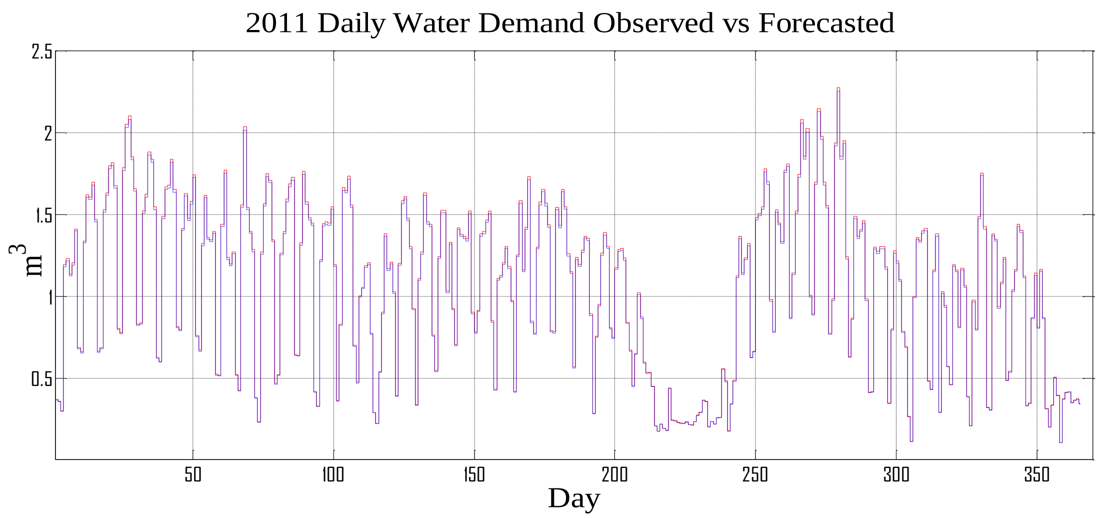



Fig. 4 Observed (red) daily and forecasted (blue) water demand

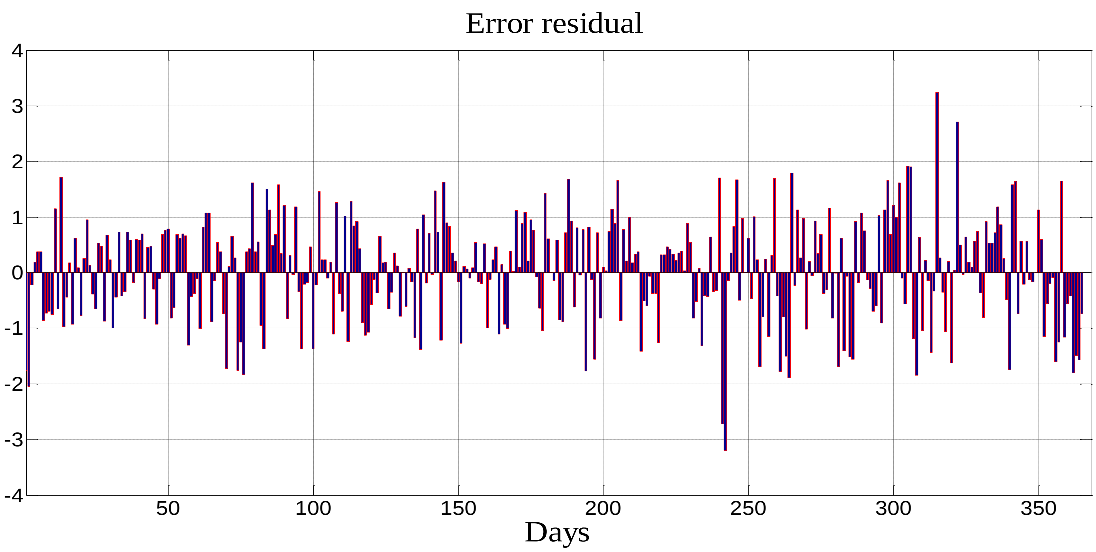



Fig. 5 2011 daily forecasted and observed normalized error residual

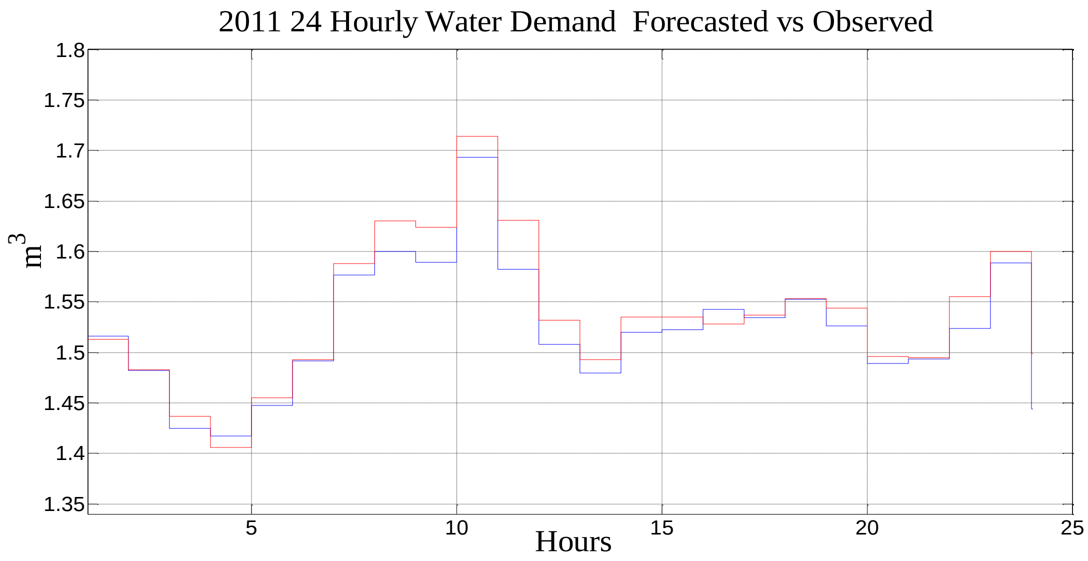



Fig. 6 Observed (red) hourly and forecasted (blue) water demand

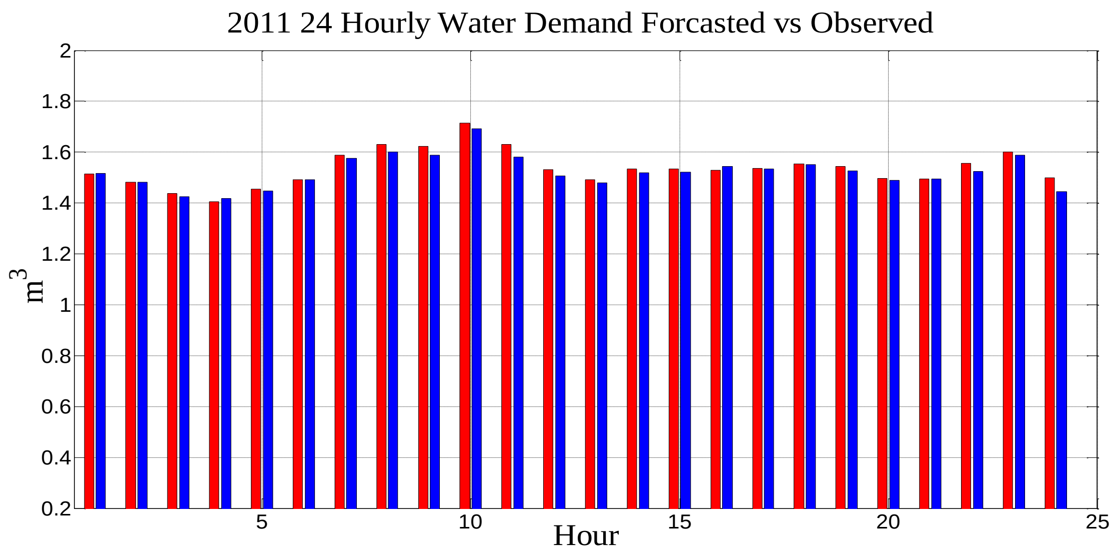



Fig. 7 Observed (red) hourly and forecasted (blue) water demand comparison

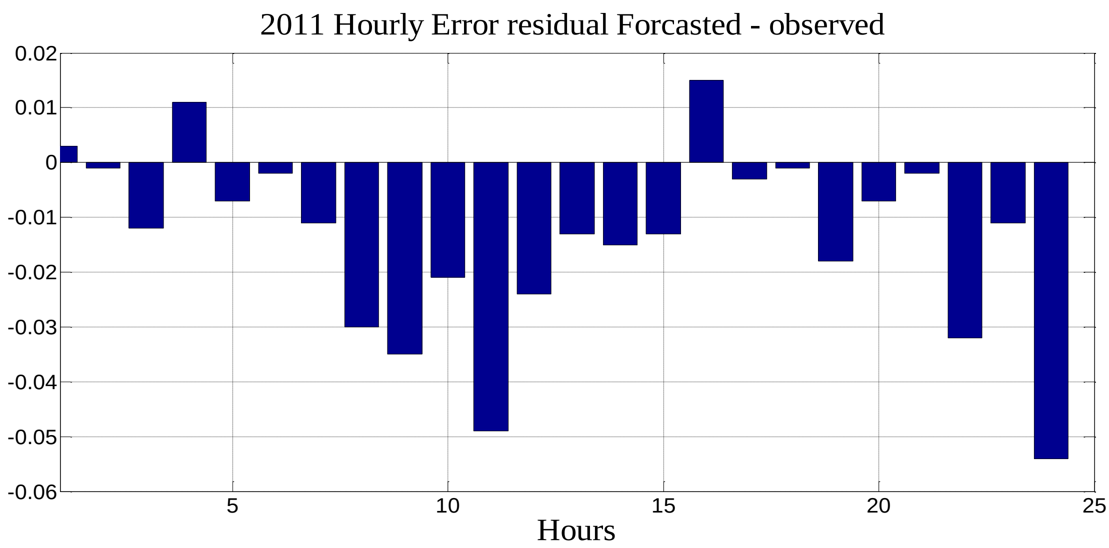



Fig. 8 2011 hourly forecasted and observed error residual

### *E. Forecasting Accuracy, Validation and Sensitivity Analysis*

The practical use of any model not only depends on the forecasted output accuracy and reliability, but also varies depends on the accuracy requirements from one water utility company to another and from application to application. For this research the validation comparisons were performed by establishing whether or not the model forecasted in 2011 water consumption output satisfactorily match with the actual observed water consumption data in the year 2011. Even if the usual practice of validation typically requires a quantitative metric to be satisfied in order for validation to be confirmed, but we can still show a graphic comparison between the model-forecasted measures of short term water demand and the actual measures data. The comparison is shown in Figure 9 below.

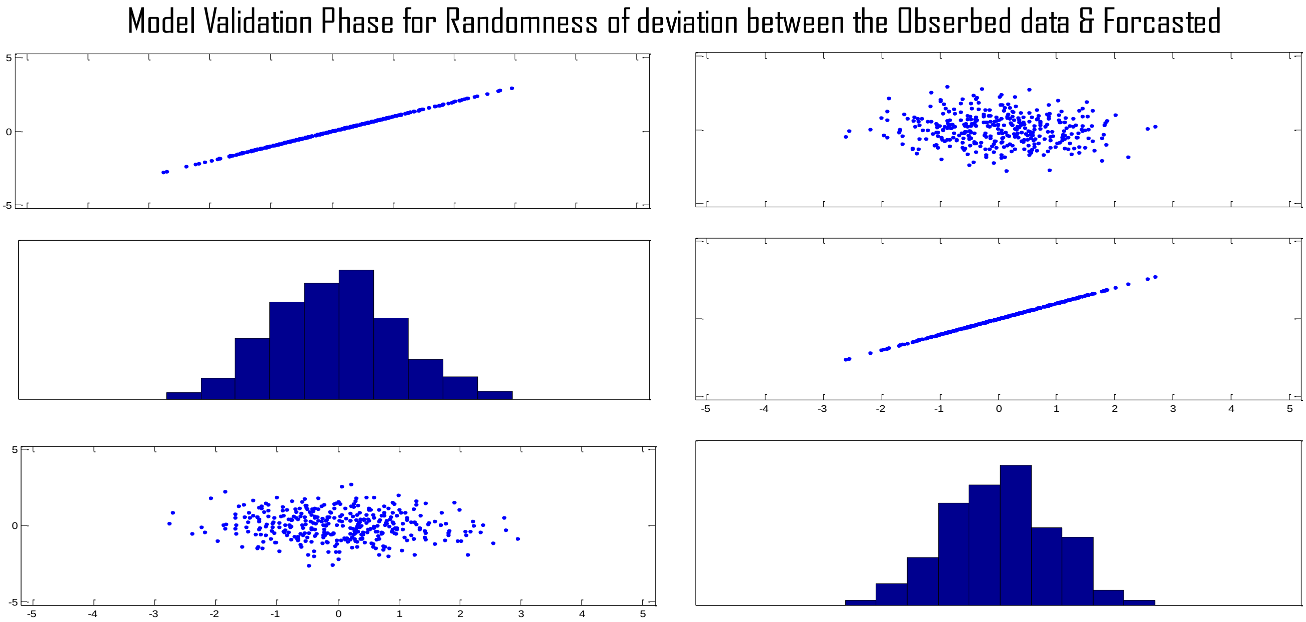
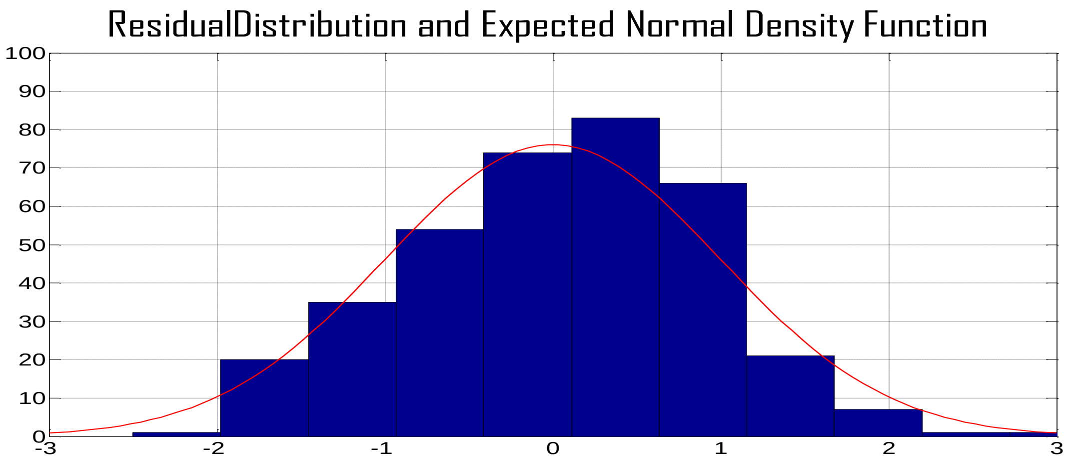



Fig. 9 Comparison between the model-forecasted measures of short term water demand

## IV. SENSITIVITY ANALYSIS

Sensitivity analysis (SA), broadly defined, is the investigation of these potential changes and errors and their impacts on conclusions to be drawn from the model [8]. In this research the sensitivity analysis (SA) was performed to test the model forecasting validity or accuracy and the forecasting model pattern and persistence components for the daily and hourly 2011 water demand forecasts, taking into account, the periodic and the periodic and persistence component together. The accuracy and sensitivity analysis were calculated and plotted as shown by both daily and hourly 2011 forecasted values. The residual error between forecasted and actual water demand is modelled as a normal distribution with mean of zero and standard deviation of the residual error results as shown in Figure 10, for these reasons the forecasting errors were measured using, Average Absolute Relative Error (AARE), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Tracking signal (TR). Tracking signal is a measure of how often our estimations have been above or below the actual value. It is used to decide



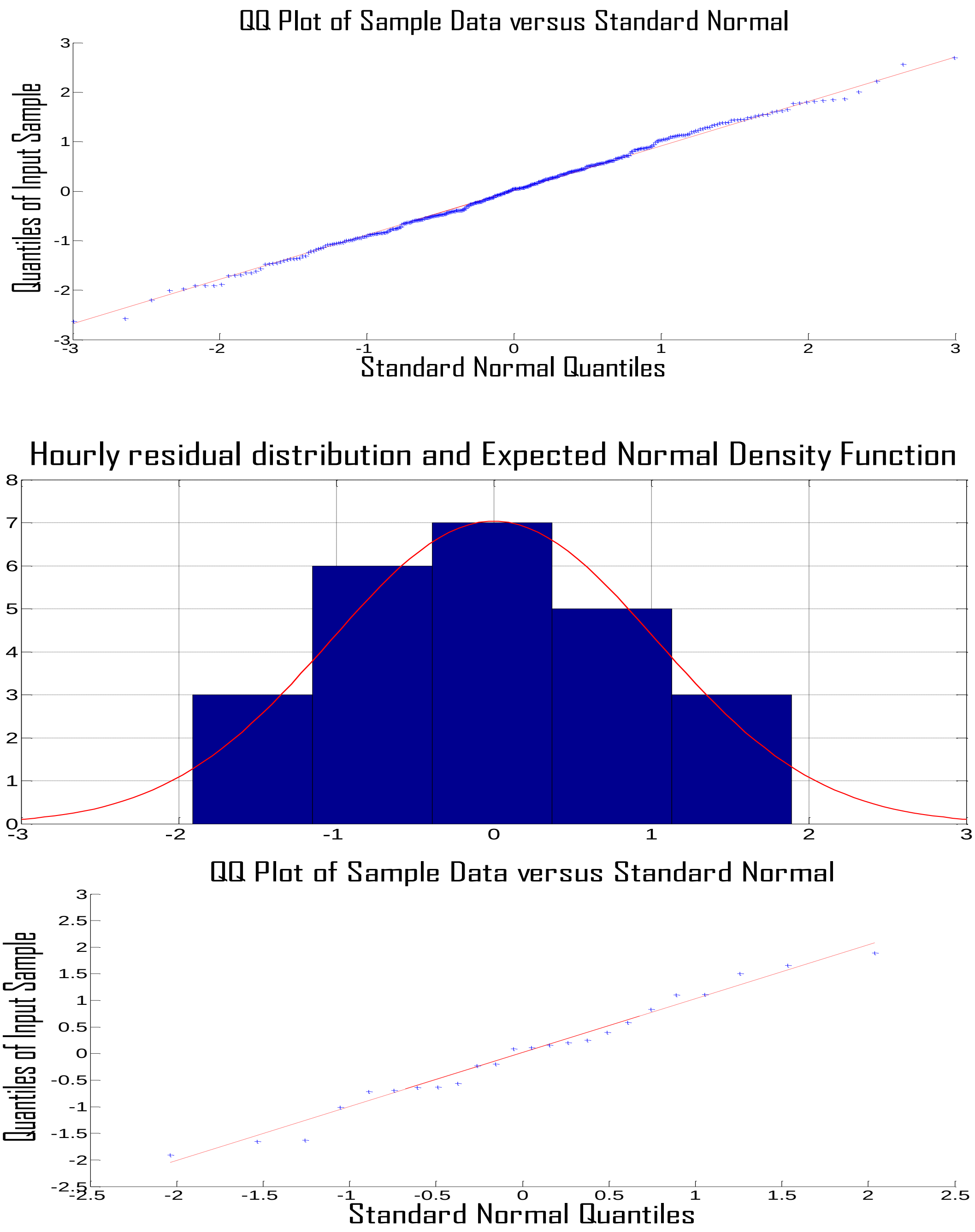



Fig. 10 The residual error between forecasted and actual water demand

where to re-evaluate using a model. Most of the Positive Tracking signal values show the actual values are above forecasted values, on other hand the Negative Tracking signal values shows the actual values are below the forecasted values, and in most of the time it ranges from 4 and -4. ( $4 \leq TR \leq -4$).

$$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n}|F_t - A_t| \tag{14}$$

The ideal MAE is zero which would mean there is no forecasting error. The larger the MAE, the less accurate the resulting model, if errors are normally distributed, then $e_\varepsilon = 1.25$ MAE. A data set with a smaller mean absolute deviation has data values that are closer to the mean than a data set with a greater mean absolute deviation.

$$\text{NPE} = \frac{\bar{e}}{\frac{1}{n}\sum_{t=1}^{n} A_t} \tag{15}$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\left[\sum_{t=1}^{n} e_t^2\right]} \tag{16}$$

$$TR = \frac{\sum_{t=1}^{n}(A_t - F_t)}{\frac{1}{n}\sum_{t=1}^{n}|F_t - A_t|} \tag{17}$$

Where $F_{i.t}$ forecast for series $(i)$ at time$(t)$, $A_{i.t}$ actual for series $(i)$ at time$(t)$, $\overline{e}_i$ average error for series$(i)$, MAE mean absolute error or deviation, NPE net percent error, RMSE root mean square error, TR tracking signal. The results of the calculation are summarized below in table 1 and Figure 11.

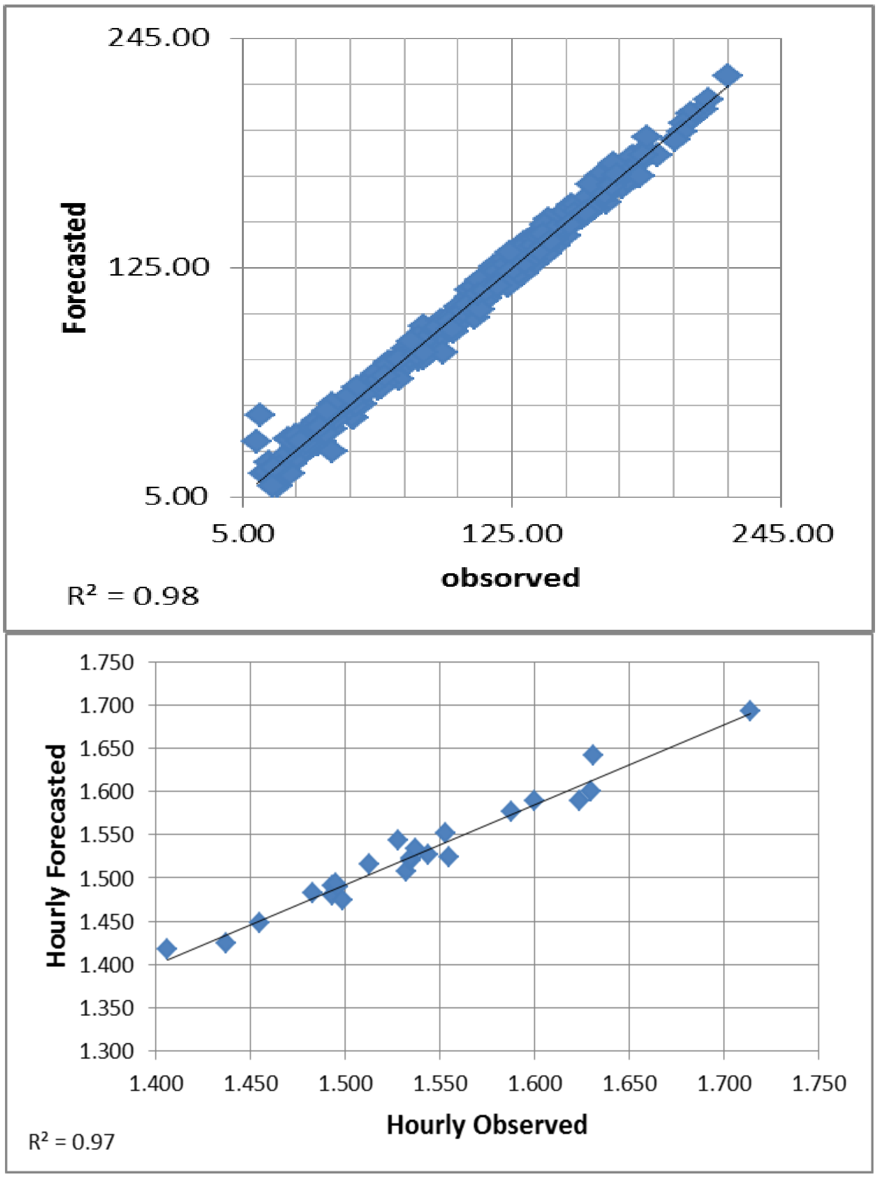



Fig. 11 comparison plot of observed versus predicted values

TABLE 1

| Results of forecasted accuracy measurement | 2011 Daily | 2011 Hourly |
|---|---|---|
| MAE % | 3.28 | 1.63 |
| MAD | 3.28 | 1.63 |
| NEP | 1.86 | 1.1 |
| RMSE | 1.86 | 1.27 |
| TR | 0.54 | 0.85 |
| CORR | 0.98 | 0.97 |
| R^2 | 0.987 | 0.97 |

The Table 1 above shows the result of accuracy measurement and sensitivity analysis

## V. CONCLUSIONS

In this paper, we presented Water Demand Forecasting Using the Stochastic Nature of Short Term Historical Water Demand and Supply Pattern, and demonstrated the data analysis results obtained and shown graphically, which confirmed that the model is doing considerably well and give reliable forecasts for daily and hourly basis water demand. The modelling results also indicate that the seasonal patterns and statistics of the generated demands trends are consistent with the actual. This approach could be easily implemented for short term water demand forecasting system to support day to day operations, as well as budgeting and financing management, and program tracking and evaluation.

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